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HYDRODYNAMICS, HEAT AND MASS TRANSFER IN A BED OF FINE NON-POROUS PARTICLES

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Аннотация—В работе приводятся результаты теоретических и экспериментальных исследований гидродинамики, тепло- и массообмена в слое мелких непористых частиц (стальных шариков, песка и щебня), выполненных при разработке нового высокоэффективного метода тепловой обработки заполнителей бетона для производства зимних бетонных работ [1].

NOMENCLATURE

- τ, time [h];
- t, temperature [deg];
- t_{wb}, wet bulb temperature [deg];
- P, pressure [kg/m^2];
- H, height [m];
- l, length [m];
- d, diameter [m];
- γ, specific weight [kg/m^3];
- S_{sp}, specific surface of granular bed [m^2/m^3];
- g, gravitational acceleration [m/s^2];
- ρ, density [kg s^2/m^4];
- a, thermal diffusivity [m^2/s];
- ν, coefficient of kinematic viscosity [m^2/s];
- λ, thermal conductivity [kcal/mh deg];
- α, heat-transfer coefficient [kcal/m^{2}h deg];
- m, void fraction;
- v, velocity [m/s];
- f, resistance coefficient;
- ψ, shape factor of bed;
- w, moisture content [%];
- T, heat content of vapour–air mixture per 1 kg of dry air [kcal/kg];
- B_0, moist air volume in a mixture per 1 kg of dry air [m^3/kg];
- r, evaporation heat [kcal/kg];
- c, heat capacity [kcal/kg].

Similarity numbers

$Eu = \dfrac{\Delta P}{\rho V^2}$, Euler number;

$Re = \dfrac{Vd}{\nu}$, Reynolds number;

$Pr = \dfrac{a}{\nu}$, Prandtl number;

$Nu = \dfrac{\alpha l}{\lambda}$, Nusselt number.

Indices

- b, bed;
- f, fluid flow or gas flow;
- 0, initial value;
- av, average value;
- fin, final value;
- tr, true;
- sp, specific;
- hyd, hydraulic;
- eq, equivalent;
- sm, smooth;
- r, rough;
- ch, characteristic;
- dr, drying.

I. THEORY

It is advisable to begin the study of the theory of heat and mass transfer in a bed with an analysis of the main principles of liquid-flow hydrodynamics within it, since the properties and character of a moving medium are the most important factors defining the convective heat and mass transfer.

The study of phenomena taking place when a liquid moves in a porous medium is one of the most complicated problems of hydrodynamics. First of all this is due to the uncertain geometry of the flow boundaries in a granular bed.

In the case of a body in a flow the geometrical property is completely defined by a linear dimension of the body whilst for a liquid flow in tubes and channels it is defined by the length and diameter of the channel, whereas the geometrical structure of a granular bed depends on the dimensions of the particles, linear dimensions of the bed and void fraction. However the whole complex of these properties does not yet define the geometry of a bed uniquely because of the casual character of the packing of grains in the filling material.

At present there is no single view as to whether the process of liquid motion in a porous medium should be referred to the internal or to the external problems of hydrodynamics. To perform calculations the resistance formula for tubes is normally applied:

$$\Delta P = f \frac{l}{d} \frac{\gamma V^2}{2g} \tag{1}$$

The use of the Navier–Stokes and continuity equations in studying fluid motion in a granular bed does not give rise to doubt, at least in the case of beds where the pore diameter exceeds the molecular mean free path. For the universal resistance law in a bed the criterial equation derived from these equations [2] by the similarity method may, therefore, be employed

$$Eu = f(Re) \tag{2}$$

The dimensionless resistance coefficient in equation (1) is related to the Euler number thus

$$f = \frac{2Eu}{l/d} \tag{3}$$

In the present paper we consider the motion of fluid in a granular bed as an internal problem of hydrodynamics, similar to the case of fluid motion in rough tubes, in order to obtain the form of a functional relation in criterial equation (2).

To simulate the process the principles of the Kozeni theory of hydraulic radius [3] and the notion of a gap, introduced into the filtration theory by Schlichter [4], are used.

The fluid motion in a granular bed with the height H and void fraction m constituted of particles of diameter d with approach velocity of the main flow v_f is replaced by the fluid motion in a rough tube of length l, diameter d_{hyd}, velocity v_{tr}, with the height of the alternative roughness protrusions K.

Passing over to the model of the phenomenon the following relations are used

$$v_{tr} = \frac{v_f}{m} \tag{4}$$

$$l = 2H \tag{5}$$

$$d_{hyd} = \frac{4m}{S_{sp}} \tag{6}$$

$$S_{sp} = \frac{6(1-m)}{d} \tag{7}$$

$$K = d_{hyd}(1-n) \tag{8}$$

In the formula (8) n is the gap, i.e. the value, characterizing the degree of contraction of the fluid passage area in the most narrow part of the pore channel. In the range of void fractions from 0·25 to 0·476 to calculate the gap we may use equation

$$n = 0{\cdot}625 m^{1{\cdot}4} \qquad (9)\ [4]$$

Substituting equations (4)–(6) into (1) we obtain the resistance formula for a granular bed

$$\Delta P = f_b \frac{\gamma}{2g} \left(\frac{v_f}{m}\right)^2 \cdot \frac{H S_{sp}}{2m} \tag{10}$$

In the laminar region the roughness, as is known, does not influence the resistance coefficient, therefore, by analogy suggested the criterial equation of the resistance of a bed in the laminar region may be written in the form

$$f_b = \frac{64}{Re} \tag{11}$$

On the basis of the experimental data by Zhavoronkov and Aerov [5] treated in accordance with (10) with respect to the constant in equation (11) the value "72" is obtained. This coincides with the present experiments and with the data of other studies [6].

For the turbulent region the reistance coefficient of a bed may be found from the equation obtained by Prandtl [7] for rough tubes on the basis of the experimental data of Nikuradze

$$f = \left[\frac{1}{1{\cdot}14 - 2lg\,(K/d)}\right]^2 \tag{12}$$

Using equations (8) and (9) for a normally encountered void fraction 0·4 we obtain from (12)

$$f_b = 0{\cdot}6 \tag{13}$$

From the experimental data [5–6] this value is 0·72–0·8.

Thus, the analogy suggested permits the expressions to be obtained for the dimensionless resistance coefficient of a bed in laminar and turbulent regions which agree fairly well with available experimental data.

To study the heat-transfer process under conditions of forced convection in a bed, the criterial equation is written in a similar manner to the ordinary equation of convection transfer

$$Nu = f(Re, Pr) \tag{14}$$

The form of the function f in equation (14) is usually determined experimentally.

Chukhanov [8] used the hydrodynamic theory of heat transfer to derive the criterial equation of heat transfer in a bed. The fluid motion in a bed being considered as the process of flow round a sphere complicated by the influence of neighbouring spheres, he obtained

$$Nu = 0{\cdot}25\ Re^{0{\cdot}86} \tag{15}$$

It is interesting to note that presenting the process of fluid motion in a bed as an external problem Chukhanov obtained the value for the exponent of the Reynolds number typical for the internal problem.

The criterial equation of "pure" heat transfer in a bed may be derived from the proposed analogy between fluid motion in a bed and in a rough tube.

To derive this equation Nunner's data on heat transfer in rough tubes cited by Eckert [9] are used. Roughness was created by rings of various cross sections fixed to the internal surface of the tube.

It was found that irrespective of the form of roughness elements the Nusselt number is a function of the Reynolds number and of the ratio of resistance coefficients.

On the basis of these data

$$Nu_r = Nu_{sm}\left(\frac{f_r}{f_{sm}}\right)^{0{\cdot}37} \tag{16}$$

For a turbulent, fully developed flow in an industrial smooth tube the resistance coefficient can be assumed constant and equal to 0·007 and the criterial equation of heat transfer may be written in the form

$$Nu = 0{\cdot}0243\ Re^{0{\cdot}8}\ Pr^{0{\cdot}33} \tag{17}$$

The resistance coefficient for a bed determined from the analogy between fluid motion in a bed and in a rough tube under turbulent conditions, is equal to 0·6.

Inserting these values into equation (16) and using equation (17) for "pure" heat exchange in a bed of spheres, yield

$$Nu = 0{\cdot}124\ Re^{0{\cdot}8}\ Pr^{0{\cdot}33} \tag{18}$$

II. EXPERIMENTAL

In the works on experimental study of heat and moisture transfer process in a bed three main methods are used to define heat- and mass-transfer coefficients:

(i) method of transient heating of a bed with the use of the Schumann analytical solution [10],
(ii) diffusion method, and
(iii) method based on determination of heat- and mass-transfer coefficients in a moist bed during a constant rate of drying.

The most accurate results are obtained by the diffusion method of local modelling [11, 12] based on simulation of the process of external heat transfer by external diffusion. This method allows the influence of longitudinal heat conductivity of a bed, internal thermal resistance of particles, heat losses through the walls of the vessel and radiant heat transfer to be excluded. The application of the method is restricted to the particles of regular shape since it is impossible to model a bed of real lumpy materials.

The method of definition of heat emission factor for a constant rate of drying of a bed [13, 14], in spite of its simplicity, may be used

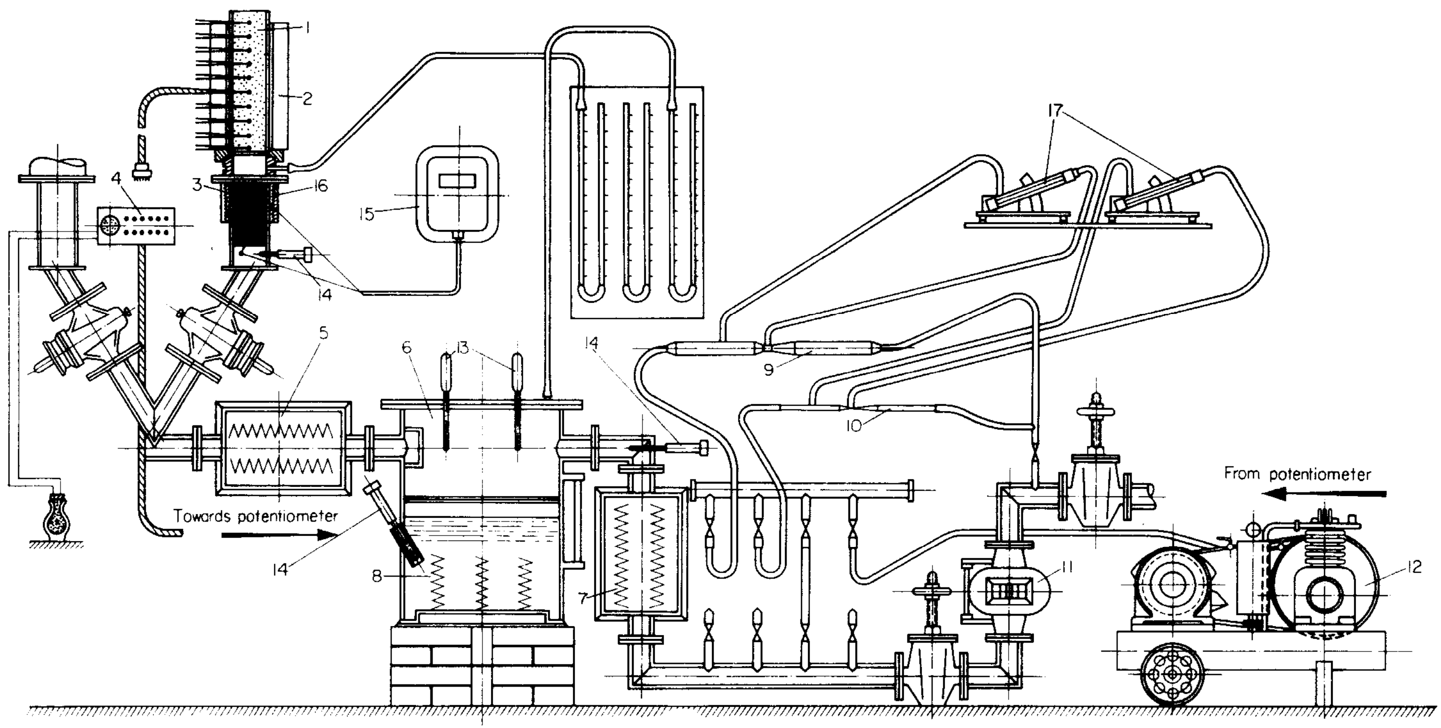

FIG. 1. Diagram of experimental unit.

1. Cylinder with material; 2. Insulation; 3. Levelling bed of metal spheres; 4. Panel for thermocouples; 5. Electric calorifier of the second order; 6. Evaporator; 7. Electric calorifier of the first order; 8. Electric heater; 9 and 10. Venturi pipe; 11. Rotational gas counter; 12. Compressor; 13. Mercury thermometer; 14. Mercury contact thermometer; 15. Correction galvanometer with differential thermocouple; 16. Electric heater of supply tube; 17. Micromanometers.

only when the particles constituting the bed have sufficient internal porosity and moisture capacity.

The method of transient heating and cooling a bed using the Schumann analytical solution [15, 16, 17] is, by its nature the only one for finding the heat emission factor in the bed of real non-porous materials.

The methods of matching experimental curves with the theoretical ones when conducting experiments on the methods of transient heating suggested by Fernes [15], Saunders and Ford [16], Chukhanov and Shapatina [17] are very laborious and do not result in great accuracy. The method developed by Aerov [11] and Vetrov [18] based on the graphical differentiation of the exit curves requires considerably less time and trouble and under certain conditions ($Re > 20$; $Bi < 0{\cdot}2$; $t_{av} < 500$) allows accurate results to be obtained.

The experimental installation (Fig. 1) consisted of a cylinder 120 mm in diameter with the material studied, apparatus for thermal-moisture heat treatment of air, a system of automatic control of heat-transfer medium parameters, a block of temperature self-recording potentiometers, exit gauges, a compressor, regulator and an instrument for measuring pressure drops.

To compare experimental data with the results of other workers and to generalize the empirical relations in the experiments, beds of metal spheres of various dimensions as well as sand and broken stone were used as a standard.

Resistance of the bed

Experiments on determination of the resistance of the bed were carried out with steel spheres, washed by quartz sand and granitic sand, and with a mixture of various compounds (Table 1).

Void fraction of individual fillings varied within the range from 0·44 to 0·48 for broken stone, 0·42 to 0·45 for sand, 0·33 to 0·37 for spheres. The height of the bed in the experiments

Table 1. *Properties of fillings*

No.	Name of material	Dimension of particles (mm)	Equivalent diameter $\sqrt{m}$	Specific weight of bed particles (kg/m³)
1	Sand	0·6–1·2	0·00109	2700
2	Sand	1·2–2·5	0·001785	2700
3	Sand	2·5–5·0	0·0028	2700
4	Mixture of sand	2·5–5·0 —20% 1·2–2·5 —15% 0·6–1·2 —22% 0·3–0·6 —41% 0·3–0·15— 4%	0·000878	2700
5	Broken stone	2·5–5·0	0·00365	2700
6	Broken stone	5·0–10	0·00648	2700
7	Broken stone	10–20	0·0105	2700
8	Steel spheres	4·7	0·0047	7800
9	Steel spheres	3·15	0·00315	7800
10	Steel spheres	7·2	0·0072	7800
11	Mixture of steel spheres	3·15—35% 4·7 —33% 7·2 —32%		7800
12	Mixture of steel spheres	3·15—20% 4·7 —20% 5·5 —20% 7·2 —20% 15·8—20%	0·0062	7800

with broken stone and sand was 300–400 mm, with steel spheres, 150–200 mm. The velocity of the flow in the supply tube ranged from 0·02 to 1·5 m/s.

The treatment of experimental data consisted of the calculation of the resistance coefficient of the bed by equation (10) and evaluation of the resistance coefficient dependence on the Reynolds number.

The hydraulic diameter was taken as a characteristic dimension, and the true velocity was calculated by formula (4).

For a bed of sand and broken stone the specific surface was determined by relation (7) but as diameter the equivalent volumetric particle diameter was inserted into it.

The results of the experiments plotted logarithmically in co-ordinates $f_b - Re$ (Fig. 2) make the following conclusions possible:

(i) Experimental data for particles of different dimensions and even for mixtures of several fractions for every material are well fitted by one curve. The curve of resistance for spheres represents the equation obtained experimentally by Zhavoronkov and Aerov [5].

(ii) The greater the difference between the form of particles and that of a sphere, the higher the curve of resistance, but all the curves in a laminar, transient and turbulent region (in the investigated range of Reynolds numbers variation from 2 to 1000) are equidistant.

To correlate experimental data on the resistance of the bed of natural materials and of spheres the shape factor was used which showed by how much the true specific surface exceeded the specific surface of the bed calculated by the

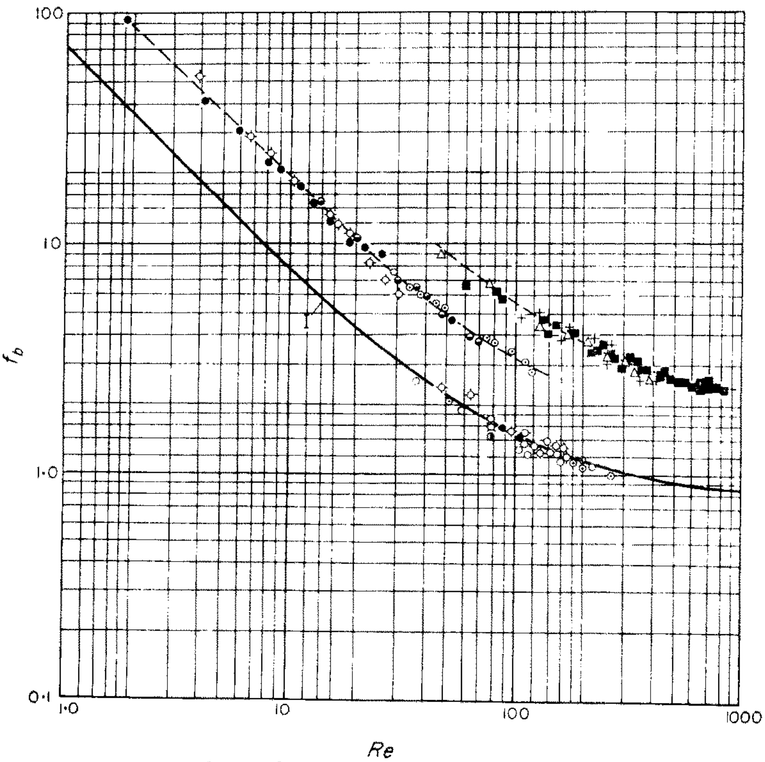

FIG. 2. Dependence of resistance coefficient on Reynolds number.

Spheres: ○ — 3·15 mm; ☒ — 4·7 mm; Mixture of spheres ◑ — 3·15, 4·7, 5·5; ⊗ — 3·15, 4·7, 5·5, 7·2, 15·8 mm; Sand ● — 0·6 to 1·2 mm; ◒ — 1·2 to 2·5 mm; ⊙ — 2·5 to 5·0 mm; Mixture of sand — ⌖ ; Broken stone △ — 2·5 to 5·0 mm; □ — 5 to 10 mm; + — 10 to 20 mm; I—$f_b = (72/Re) + 0{\cdot}8$.

value of the diameter equivalent volumetric diameter.

The shape factor was determined from experimental data by the iteration method.

Various values of the shape factor ψ were substituted into the formula

$$S_{sp} = \frac{6(1-m)\psi}{d_{eq}} \tag{19}$$

for determination of the specific surface of the bed. Those values of the factor ψ which allow the resistance curve for the given material to fit well to the resistance curve for spheres were considered true.

The values found in this way are: 1·6 for sand and 2·4 for broken stone.

On the graph, representing the results of all the experiments, the relations and experimental equation of Zhavoronkov and Aerov [5] for particles of regular shape are plotted.

Heat transfer in a bed

Experimental investigations of heat transfer in a bed of steel spheres 3·15; 4·7; 5·5; 7·2; 15·8 mm in diameter, of sand of fractions 1·2–2·5; 2·5–5·0, and of broken stone of 2·5–5·0; 5–10; 10–20 fractions, were carried out by the methods of transient heating using the theoretical solution of Schumann. To determine the heat emission factor the method of graphical differentiation of exit curves was used [11, 18].

Experiments were carried out in a cylinder 120 mm in diameter and 150 mm in height during cooling and heating. In heating, air at 100–150° was blown through the bed. The air velocity in the supply tube ranged from 0·15 to 1·7 m/s.

When analysing experiments the characteristic dimension and velocity of the flow were assumed the same as in studying the resistance of the bed.

The characteristic temperature was calculated by the formula

$$t_{ch} = \frac{t_{b(0)} + t_{f(0)}}{2} \tag{20}$$

The experimental data over the investigated

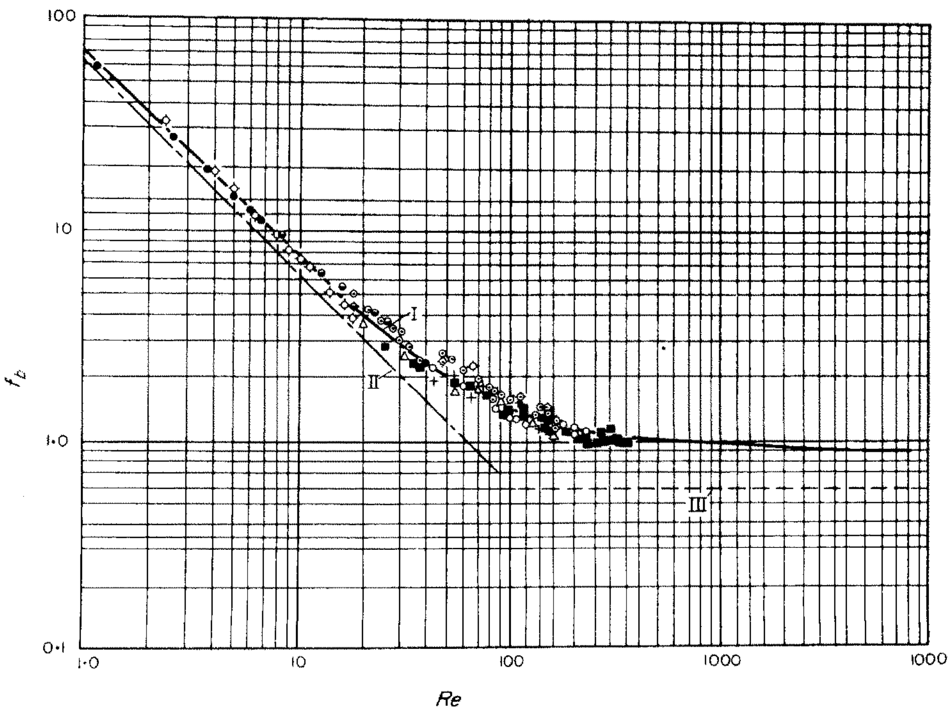

FIG. 3. Matching of experimental data on resistance of bed of steel spheres, sand and broken stone I—$f_b = (72/Re) + 0{\cdot}8$; II—$f_b = (64/Re)$; III—$f_b = 0{\cdot}6$.

Reynolds number range (20–1000) for sand and steel spheres of various fractions and dimensions within the limits of experimental accuracy, are fitted in the logarithmic plot by one line of which equation is

$$Nu = 0{\cdot}11\ Re^{0{\cdot}86}\ Pr^{0{\cdot}33} \tag{21}$$

The points corresponding to the experiments with broken stone lie higher than this straight line, but parallel to it. The shape factor, equal to 1 in this case for sand and for broken stone to 1·5, being introduced, experimental data of the experiments with broken stone agree with the data on heat transfer in a bed of spheres and sand (Fig. 4).

Thus, the shape factors of a bed for broken stone and sand determined from resistance and heat transfer curves are not the same, and only the ratio between them equal to 1·6 remains constant.

The amount of the experimental material obtained is not enough to explain and substantiate this fact, since only two shapes of natural particles with various surface shapes were used in the experiments.

First of all we may assume that there is a considerably greater amount of stagnant zones with closed circulation in the bed of natural materials than in the bed of spheres. The surface of these zones which contribute to the hydraulic

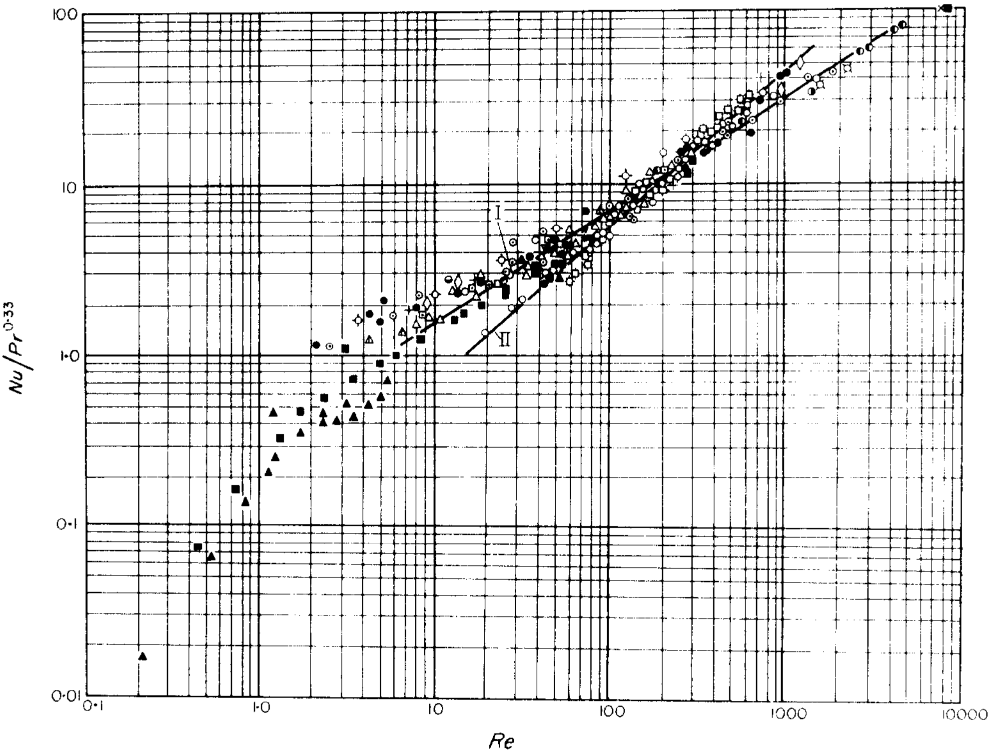

FIG. 4. Dependence of dimensionless heat-transfer coefficient in bed on Reynolds number. Aerov—steel spheres ⌖ 3·19; ● 7·15; ⊙ 19·35; Catalysing pellets ◬ —*d* 6·65 *H* 6·95; ◊ *d* 9·1 *H* 10·2; Rashig rings ⊡ — *d* 8 *H* 8 mm; Bar-Ilan and Resnick ■ pellets 9·5 × 6·8; △ 4 × 4; granulars ▲ 0·084 mm. Tecker and Hougen, Rashig rings ○ — *d* 13, *H* 13; ◐ — *d* 25, *H* 25; ◑ — *d* 38, *H* 38 mm. Shapatina -●- steel spheres *d* 4·75. Semenov steel spheres ⌖ 7·92. Authors' experiments, steel spheres ✦ — 15·8; ○ — 7·2; ⊕ — 5·5; ⊖ — 4·7; ⌘ — 3·15 mm; quartz sand ⊠ — 1·2 to 2·5; ✱ — 2·5 to 5·0 mm; crushed granite + — 10 to 20 mm; ⌗ — 10 to 5 mm; ◬ — 5 to 2·5 mm. I—$Nu/Pr^{0·33} = 0·395\ Re^{0·64}$; II—$Nu/Pr^{0·33} = 0·11\ Re^{0·86}$.

resistance, does not take part in heat transfer.

The results of our experiments and of other investigations on heat transfer in a bed composed of particles of regular shape and various configurations (spheres, Rashig rings, catalysing pellets, granular) carried out by the diffusion method [11, 12], method of transient heating of bed [17] and drying of moist bed at constant rate [14] are treated by one method and given in Fig. 4.

The data obtained allow the conclusion to be drawn that with forced convection in a bed the laminar region too preserves the relation $Nu = f(Re)$. The lower limit of the validity of this relation is not yet found, in any case it exists up to Reynolds numbers 0·1–0·2.

The equation

$$Nu = 0{\cdot}395\, Re^{0{\cdot}54}\, Pr^{0{\cdot}33} \qquad (22)\ [11]$$

describes experimental data on heat transfer in a bed with sufficient accuracy ($30 < Re < 5000$) obtained by the diffusion method with constant rate of drying with simultaneous heat and mass transfer.

The results of experiments on "pure" heat transfer obtained in the process of steady and transient heating do not obey this relation, and in the range of Reynolds numbers (20–1000) are described by equation (21) which differs slightly from equation (18) obtained from the analogy between the fluid motion in a bed and in a rough tube.

It is important to note, that in all the works known dealing with the determination of the heat-transfer coefficient in a bed with "pure" heat transfer [15, 16, 17] the value of the exponent of the velocity in the empirical formulae determining heat-transfer coefficient in the range of 0·7–1·0 and on the average is 0·85.

With heat transfer stimulated by mass transfer and with simultaneous heat and mass transfer the value of the exponent of the velocity is on the average 0·63 [11, 12, 14].

Heat and mass transfer in a bed of moist sand

To study the process of heating a bed of moist frozen-together non-porous particles, washed quartz sand was used dispersed in fractions of 0·6–0·3; 1·2–0·6 mm.

The experimentation was based on recording temperatures of the bed and heating medium when blowing hot air through sand. For this purpose cylinders 120 mm in diameter and 450 mm in height were used with thermocouples set at different height.

The experiments were carried out in the following succession: dry sand at room temperature was wetted with water up to the required degree (3–6% by weight) and a cylinder was filled with it. Samples of the material (3–5 pieces) were packed and then trial samples were taken to determine the initial moisture content.

Then the cylinder with sand was weighed, hermetically sealed from both ends and placed into a refrigerator where it was kept at a temperature of −50° for 18–20 h. Removable insulation was placed into the refrigerator as well.

By means of the automatic control system the necessary regime characteristics of the experiment: viz. flow rate, moisture content and air temperature were set, and the apparatus was set in motion. Then the cylinder with the frozen sand was taken out from the refrigerator and weighed, with the blower switched off, was screwed on to the supply tube. The cooled insulation was put on it from the outside, the thermocouples connected with the self-recording potentiometer, the blower switched on, and the experiment started.

The experiment was over when the temperature of the heating medium leaving the bed became equal to the temperature of the air flow at the inlet into the bed. Then the cylinder was taken away, weighed and the samples were taken from it to determine the final moisture content of the material.

The duration of each experiment ranged from 5 to 15 h depending on the velocity and moisture content of the air flow. The temperature of the air supplied to the bed was kept constant and equal to 95°. It was only its moisture content that varied. The experiments with 0·6–0·3 mm sand were carried out under three operational conditions with air wet-bulb temperature 32°, 44°, 60°, and the experiments with 0·6–1·2 mm were carried out under two operational conditions, 32°, 44°. The air velocity in the supply tube varied from 0·1 to 0·25 m/s.

It was determined from the analysis of the primary experimental material, thermograms, taken by a self-recording potentiometer (Fig. 5) that, as distinct from the temperature curves of heating of dry material, the thermograms of the moist bed have a horizontal area caused by the phase transformation in the bed, that is, evaporation. This area, independent of the initial temperature of the bed, appears at wet-bulb temperature.

The validity of this fact was verified for the case when the initial temperature of the bed is higher than the wet-bulb temperature. For this purpose a number of test runs were carried out with sand of fraction 0·6–1·2 mm, which in a dry state was heated in an air thermostat up to 60°, then wetted with hot water, and the air with dry-bulb temperature 95° and wet-bulb temperature 30° was blown through it. An interesting phenomenon was observed here (Fig. 6). Firstly, the hot sand with hot air blown through it was cooled up to the wet-bulb temperature and then was heated again up to the initial dry-bulb temperature of the air.

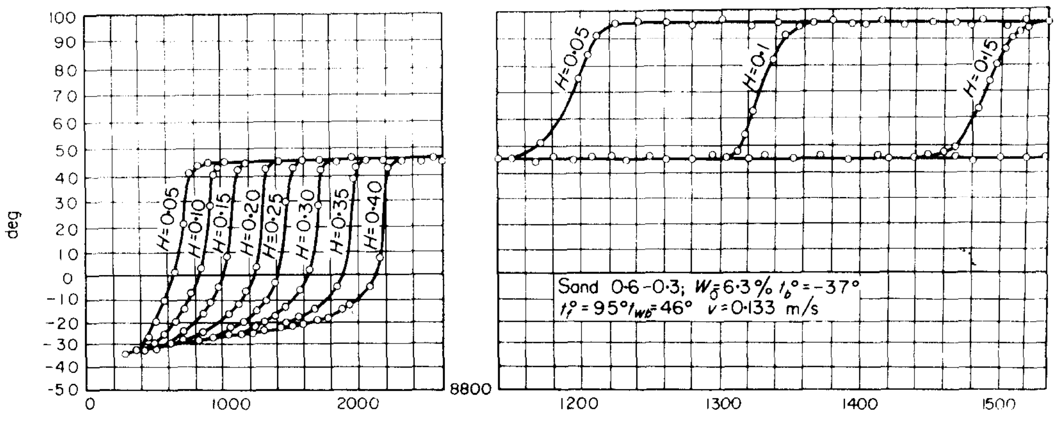

FIG. 5. Thermogram of heating of moist sand 0·6–0·3 mm.

$t_{wb} > t_{b(0)}$

$t_{b(0)} = -37°$; $W_{b(0)} = 6·3\%$; $t_{f(0)} = 95°$; $t_{wb} = 46°$; $v_f = 0·133$ m/s.

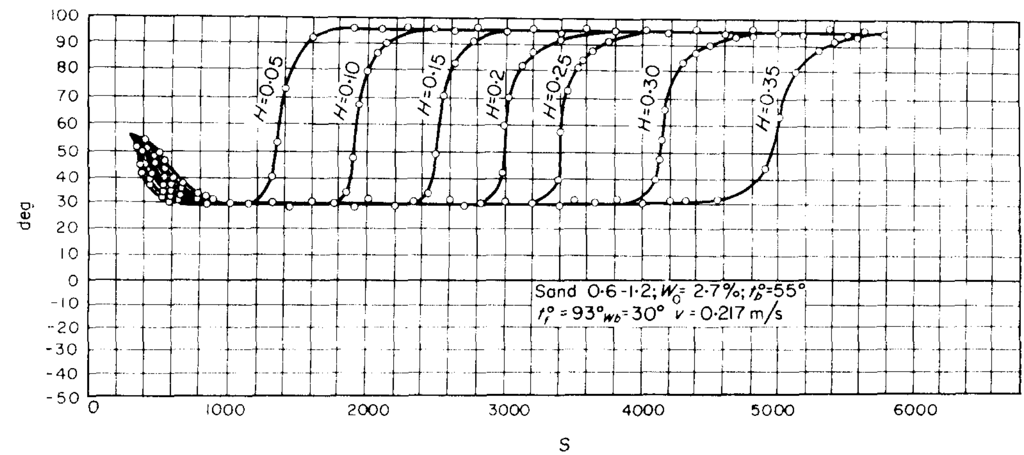

FIG. 6. Thermogram of heating of moist sand 0·6–1·2 mm.

$t_{wb} < t_{b(0)}$

$t_{b(0)} = 55°$; $W_{b(0)} = 2·7\%$; $t_{f(0)} = 93°$; $t_{wb} = 30°$; $v_f = 0·217$ m/s

The data obtained permit the conclusion to be drawn that moist sand is heated by successive advancement through the bed: (1) the heating or cooling front up to the wet-bulb temperature, and (2) drying and heating front up to the initial temperature of the air flow. In the bed the processes of pure heating, evaporation and condensation take place simultaneously. Moisture evaporating in lower strata, condenses in upper ones speeding up their heating up to the wet-bulb temperature.

In the test runs there was a clear-cut demarkation of the drying front between the dry heated sand and moist sand at wet-bulb temperature. The experiments carried out specially on aeration of moist sand in glass tubes allow this phenomenon to be visually observed since the moist and dry sand have different colours. Perhaps under the experimental conditions the frontal surface of a bed may be considered as an evaporation surface, being on this ground the interface between dry and fully heated material and the moist material at wet-bulb temperature.

The results of experiments were treated in the form of generalized graphs with the use of dimensionless variables: relative wet-bulb temperature of a bed

$$\theta_{wb} = \frac{t_b - t_{b(0)}}{t_{wb} - t_{b(0)}} \tag{23}$$

relative dry-bulb temperature

$$\theta_{db} = \frac{t_b - t_{wb}}{t_{f(0)} - t_{wb}} \tag{24}$$

and the homochronous criterion

$$Ho = \frac{v_f}{H/\tau} \tag{25}$$

Assuming H to be the horizontal co-ordinate of the bed with temperature t_b, we may represent the homochronous criterion (25) as the ratio of two velocities: flow velocity in the bed and the velocity of motion of the bed heating front up to t_b.

Considering the dimensionless heating graphs (Fig. 7) from this viewpoint, we may establish

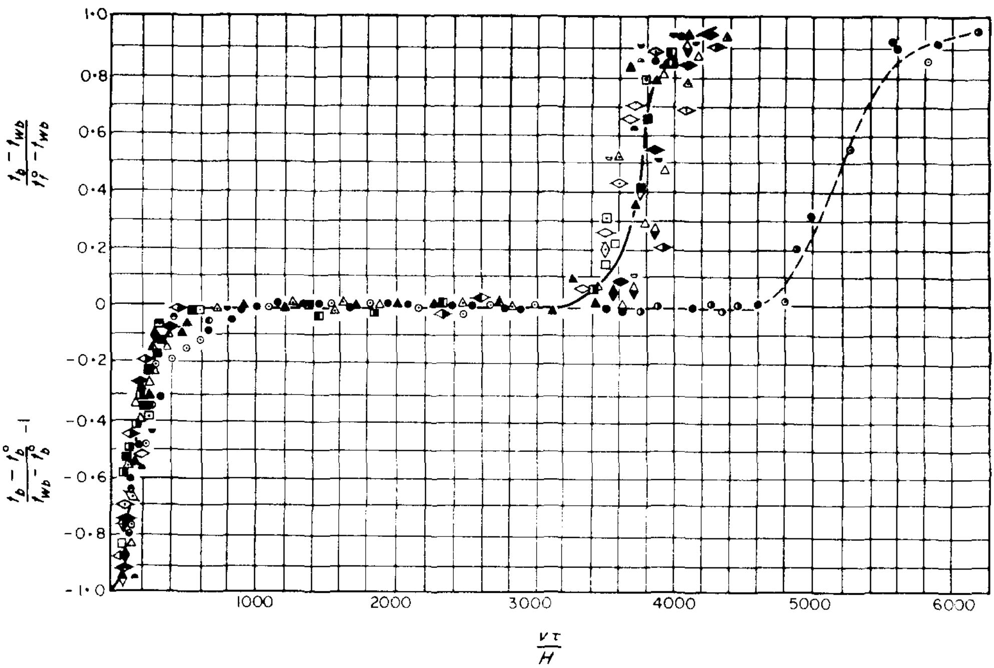

Fig. 7. Generalized graph of heating of moist sand 0·6–1·2 mm.

$t_{b(0)} = 53°$; $W_{b(0)} = 5{\cdot}3\%$; $t_{f(0)} = 95°$; $t_{wb} = 30°$;

v_f ⊙ — 0·127, ◑ — 0·164; ● — 0·20 m/s; H ⊙ — 0·05; △ — 0·1; ⊡ — 0·15; ◇ — 0·20; ◊ — 0·25; ▽ — 0·30; ⌓ — 0·35 m.

that for all the experiments the points corresponding to various heights of the bed (beginning from 0·05 m) are fitted by one curve, and heating up to $\theta_{wb} = 1$, $\theta_{db} = 1$ ceases at one and the same value of the number H_0 at every height. Consequently, the velocity of advancement of the heating front up to the wet-bulb temperature, of the drying front and complete heating of the bed is for the given conditions ($H/d > 100$) a constant value. With the rise of the air-flow wet-bulb temperature the velocity of advancement of the heating front up to t_{wb} increases and that of the drying one decreases.

From the analysis of dimensionless graphs simple relations for calculation of the heating and drying rates were obtained.

With the initial temperature of the bed less than zero the velocity of advancement of the heating front up to the wet-bulb temperature is determined by the formula

$$v_{wb} = v_f \frac{T_0 - T_{fin}}{A \cdot B_0 \cdot \gamma_b \, [C_b \, (100 - W_b/100) \, (t_{wb} - t_{b_{(0)}}) + (W_b/100) \, (-0{\cdot}5 \, t_{b_{(0)}} + 80 + t_{wb})]} \tag{26}$$

whilst that of advancement of the drying front and complete drying of the bed by the formula

$$v_{dr} = v_f \frac{(t_{f_{(0)}} - t_{wb}) \cdot C_f \cdot \gamma_f}{A \cdot \gamma_b [C_b \, (100 - W_b/100) \, (t_{wb} - t_{b_{(0)}}) + (W_b/100) \, (-0{\cdot}5 \, t_{b_{(0)}} + 80 + t_{wb} + r)]} \tag{27}$$

With the initial temperature higher than zero the denominator of relations (26), (27) loses the terms involving heat consumption for heating and defrostation of ice.

The factor A is determined experimentally. On the basis of the data we have for sand it may be assumed equal to 1·25.

The final heat content of air in formulae (26) and (27) is taken at a temperature equal to the initial temperature of the bed and at complete saturation.

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Abstract—This paper deals with results of theoretical and experimental studies on hydrodynamics, heat and mass transfer in a bed of fine non-porous particles (steel spheres, sand and crushed stone). The analysis was carried out during the development of a new, highly effective method of thermal treatment of concrete aggregates for applying concrete under winter conditions [1].

Résumé—Cet article traite des résultats des études théoriques et expérimentales sur l'hydrodynamique, le transport de chaleur et de masse dans un lit de fines particules non-poreuses (sphères d'acier, sable et pierre écrasée). L'analyse fut conduite pendant le développement d'une nouvelle méthode de grande efficacité pour le traitement thermique d'aggrégats de béton afin d'employer le béton dans les conditions hivernales [1].

Zusammenfassung—Die Ergebnisse theoretischer und experimenteller Untersuchungen der hydrodynamischen Vorgänge und des Wärme- und Stoffüberganges in einer Schüttung aus feinen, nichtporösen Teilchen (Stahlkugeln, Sand- und Steinschotten) werden angegeben. Die Untersuchung erfolgte während der Entwicklung einer neuen, sehr wirksamen Methode zur Wärmebehandlung von Betongemischen, um eine Verwendung des Betons auch unter winterlichen Bedingungen zu ermöglichen [1].